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Development of Statistical Model for Prediction of Dynamic Modulus of Cement Concrete of Known Mixed Design

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Abstract—Dynamic modulus (E) of concrete is an important strength parameter used in the design of many civil engineering structures. It is greatly influenced by variations in mixed design, curing condition, construction practice, and time [2].
The major objective of this study was to formulate statistical model to predict the dynamic modulus by using information about the above mentioned variation. For this purpose 6x12 inch cylindrical concrete samples of different mixed design were made and tested at different ages by the impulse load test. The data obtained was used for the development of this model that provides an indirect nondestructive testing procedure. The developed model is helpful to eliminate undue construction delays of newly constructed structural members. Moreover, the model is useful for reaching a mixed design quickly without making and testing many samples of differently mixed proportions, as recommended by ACI Standard 211.1.

Index Terms—

1 Introduction

THE primary objective of this study was to develop statistical model relating dynamic modulus with mixed design variables. In this work a total of 144 concrete samples of 6x12 inch size were prepared and tested at different ages. A data base comprised of 528 data points was obtained from this work that was utilized in the development of this model. For this purpose multiple linear regression (MULTR), numerical optimization (NUMOPT), and Statistical Analysis Software (SAS), were used. The first two programs were written by McCuen (1992), whereas SAS is popular software available on the market.

In the development of regression model, the first attempt is normally made to fit the data with a linear analysis because linear models are easy to apply and their statistical reliability can be assessed easily. Linear models may be rejected, however, for a number of reasons, theoretical or empirical. The reason might suggest a non-linear model structure. Non-linear models may be classified as a power, polynomial, exponential, or some other complex model structure.

In this work, a variety of different model structures were tried and prediction equations developed and compared for rationality and reliability. The best amongst them was then selected for the final model.

2 Experiment Design

This research was basically conducted to formulate the statistical model to predict the dynamic modulus by using information about the mixed design, curing condition, construction practice, and age. For this purpose, a total of 144 concrete samples of 6x12 inch were prepared and tested. The same size is recommended by ASTM for compressive strength (C31) and can also be used for dynamic modulus calculation [1]. In these samples a wide variation in properties was accomplished either by changing the mixed design, such as the coarse fine aggregate ratio (C/F), aggregate cement ratio (A/C), water cement ratio (W/C), maximum size of coarse aggregate (Max), curing time (Cur), or age of the samples. In fact, these are the variables that have an effect on concrete strength, as reported by Akhtar (1988). All six predictor variables were used in designing the experiment of this study.

The main objective of the experiment design in this study was.

(1) Collect enough data points (N).
(2) Select the range of values for each predictor variable.
(3) Select the number of points in each range.
(4) Avoid inter-correlations between different predictor variables.

The number of data points (N) plays an important role in the formation of a statistical model. The required number of data points depends upon many factors, such as replication effect, number of predictor variables, and number of points in each selected range of values. In a preliminary study [2], it was found that the impulse load test gave the same results for a sample tested many times at one age even by different operators. Similarly, many samples made up of the same mixed gave the same results. Under these circumstances 144, concrete samples prepared in this study were tested at different ages in such a way that a total of 528 data points were obtained.

The next important step in the experiment design was to select the range of values for each of the predictor variables. In selecting these ranges, the main concern was to cover the maximum range of values possible within which that specific variable could occur. However, this was not always the case, because of experimental or procedural limitations that played an important role in selecting this rang. As an example, according

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to ASTM the maximum possible size of coarse aggregate for 6x12 inch samples should not be greater than 2 inches. However, this is not the maximum possible size for coarse aggregate. At this point, it is important to note that in coarse aggregates each maximum size contains several other smaller sized aggregates. The selection of these aggregates was made according to ASTM procedures. For this purpose all the coarse aggregates were sieved into different sizes such as 1.5 inch, 1.0 inch, 0.75 inch, 0.375 inch, #4, #8, and #16. All of these sizes were weighted separately in such a way that the ASTM requirement for coarse aggregate was met. Also, plotting the gradation curves the values of Cc (coefficient of concavity) ranged from 1.436 to 2.136 and the value of Cu (coefficient of uniformity) was close to 4 and thus well-graded requirements were fulfilled also. For the other variables, the same criteria for selecting the range of values were used. Table 1 gives the selected range of values and other points in that range for all six predictor variables.

TABLE 1
SELECTED RANGE AND OTHER POINTS FOR ALL PREDICTOR VARIABLES

PREDICTOR	RANGE	POINTS
OF	1 to 3	1, 2,3
A/C	3 to 5.5	3, 4 ,5.5
W/C (%)	48 to 75	48, 55, 62, 75
Max	0.5 to 1.5	0.5, 0.75, 1 , 1.5
Cur	2 to 12	2, 5 , 12
Age	4 to 56	4, 7, 14, 28, 37, 50, 56

The last criteria in designing the experiment of this study was to avoid the inter-correlation's between predictor variables. In fact, this was the most important criteria that could affect the rationality of the model. As an example, consider two predictor variables Xi and X2- Let the selected range for Xi be 2 to 8, and for X2 be 100 to 175. If 2,4,6, and 8 are the selected points in the range of Xi and 100,125,150, and 175 are the selected points in the range of X2, the grid can be made. This grid is also known as a fractional.

In an experiment design, if we select the values of xj and X2 exactly on the firm line, the inter-correlations between Xiand X2 will be unity. It can cause irrationality in the final model. On the other hand, if we select all the combinations of Xi and X2, i.e., the values at each intersection of the dotted line, our model will be rational. Moreover, as we increase the number of observations on each intersection point, the accuracy of the model will be increased [1].

Using the same principle, such factorials for all possible combinations of predictor variables were prepared. In each factorial 528 data points were used. These data points were obtained by testing 144 concrete samples at different ages. It is important to note that initially all 528 data points were divided equally at all intersections of dotted lines, which is ideal. However, later by using the back and forth process, some small modifications to this ideal situation wefe made. The reason for these changes was the expectation of achieving minimum and maximum possible values of the criterion variable. The factorials for all possible combinations of predictor variables with these changes.

3 SAMPLE PREPARATION AND TESTING

All concrete samples of 6x12 inch were prepared according to ASTM procedures. Single use moulds recommended by ASTM C470-8 1 were used. The maximum size of coarse aggregate used in all of the samples was according to ASTM C192, which describes that the diameter of the samples shall be at least three times the maximum nominal size of the coarse aggregate. All of the samples were made according to the ASTM C 192-81 procedure. For testing of these samples impulse load test was selected due to following reasons:

- Test method is simple and quick to perform (Akhtar, 1988).
- It predicts the effect of variables that can affect the proties of concrete (Akhtar, 1988).
- It has been used successful to detect and locate flaws and cracks in concrete samples (Carino, 1986).
- It is currently being adapted for field use (Carnio, 1986).
- Its accuracy can be checked by the coherence function (Akhtar, 1988).
- It can test larger structures by using larger sized hammers.

Testing arrangement of Impulse Load Testconsists of a hammer, an accelerometer, and a Fast Fourier Transfer (FFT) signals analyzer. In this arrangement an impulse load is produced by striking the surface of a concrete sample with a hammer and response is measured in either the time domain or the frequency domain. However, the responses from the frequency domain are more consistent [2] and therefore, were used in this work.

In the frequency domain, the resonant or the natural frequency of the concrete sample is determined from the peak value of either the transfer function or power spectral density function [2]. This resonant frequency is then used to calculate the dynamic modulus. For longitudinally vibrated samples having free-free condition, the following equation can be used [5]:

= 4(y/g)fn2L2

Where

fii = natural frequency of the sample for the first mode of vibration (Hz)

L = length of the sample (inch)

g = acceleration due to gravity (ft/sec2)

y = unit weight of the sample (lb/ft3)

4 DEVELOPMENT OF MODEL

The main purpose of this study was to develop a model that relate thedynamic modulus from the impulse load test with the mixed properties, curing condition, and age of the concrete. The data base used for the development of this model had 528 data points.

For the development of this model both linear and nonlinear model forms were tried (Table 2). Among these models

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the one represented by A-l was developed using the MULTR program. This model is linear in nature and was checked for rationality and reliability.

As for as rationality of a model is concerned it can be checked from its value of determinant of correlation matrix and standardized partial regression coefficient (t). Table 3 shows the corresponding correlation matrix and the value of the determinant. Since this value is 0.51, the developed model seems to be rational. Table 4 shows the values of't' and the corresponding R2 for this equation. Since these values of 't' are less than 1.0 in absolute value, the equation does not show any irrationality.

TABLE 2
SUMMARY OF REGRESSION MODELS FOR MODEL

MODE NO.	SOFTWARE USED	MODEL STRUCTURE	R^	Se/Sy	RATIONALITY	COMMENTS
A-l	MULTR	E=70 1 4663+3 851. 05 l*Age+ 16393. 3 1*C/F +28451 1.8*A/C-3769.28*W/C+1 23593 *Max+25923.58*Cur	0.858	0.379	Rational Model	Over Prediction At Lower Values i
A-2	MULTR, SAS, NUMOP	E=67817520*Age$^{0-031042*}$C/F^{0014707*} A /p0.236885*Tif/p-0.76088(>6:|:»,r 0.020349 *Cur^{0013252}	0.854	0.385	Rational Model	Over Prediction ! At Lower Value
A-3	SAS, NUMOP	E=1807328$^{*s-*0198482*}$Age^{00309642*} /~i /r-'O.Ol 5272* a //~*0 -2 ^80()7 *ti t //~i0.41 8062* 1 /In AY 1 *r Mf /I * Max00201279* Cur0014178	0.860	0.376	Rational Model	Over Prediction i At Lower Value I i
A-4	SAS	£=55878 ioo*(l-e-^{*557799*Aac})*C/F^{0013507*} $_{A/c}$0.237H07*$_{w/c}$^).680774*$_{Max}$O.OI9768 *$_{Cur}$0.0.273	0.866	0.369	Rational Model	Over Prediction At Lower Value
A-5	MULTR, SAS	E=40553020+0.9349872E9*[Age/(1 - 28* Age)]+ 1 6436.5*C/F+2846 1 0.9* A/ 62808.44* W/C+ 1 201 80.3*Max+ 23530.82 * Cur	0.872	0.359	Rational Model	Some Improvement in Over Prediction '

* All models are unbiased

TABLE 3
CORRELATION MATRIX AND ITS DETERMINANT FOR MODEL A-1

PREDICTOR VARIABLE	AGE	C/F	A/C	W/C	Max	Cur	E (I.L.T.)
AGE	1.000	0.000	0.000	-0.001	-0.018	0.681	0.198
C/F	0.000	1.000	0.000	0.000	0.000	0.000	0.018
A/C	0.000	0.000	1.000	0.000	-0.004	0.000	0.402
W/C	-0.001	0.000	0.000	1.000	-0.014	0.154	0.801
Max	-0.018	0.000	-0.004	-0.014	1.000	0.025	0.059
Cur	0.681	0.000	0.000	0.154	-0.025	1.000	0.092
E(I.L.T.)	0.198	0.018	0.402	-0.801	0.059	0.092	1.000

Determinant = .512

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TABLE 4
STANDARDIZED PARTIAL REGRESSION COEFFICIENT (T), R AND R^2 FOR MODEL A-1

PREDICTOR VARIABLE	T	R	R^2
AGE	0.0905	0.0393	0.1983
C/F	0.0178	0.0003	0.0178
A/C	0.4021	0.1616	0.4019
W/C	0.8247	0.6419	-0.8012
Max Cur	0.0546	0.0034	0.0587
	0.1581	0.0084	0.0916

The model was checked for its reliability using the value of R2, the standard error of estimate (Se), and the degree to which the assumptions for residual analysis were met. The value of R2 is applicable for the linear model form only. However, for non-linear model forms the corrected value of R2 was computed in this study. The value of R2 for this linear model form was 0.86.The second factor for assessing the reliability is the standard error of estimate. This factor can also be viewed in the form of Se/Sy. The value of Se/Sy for this equation was 0.38.

The plots for constant error varieties and their distribution for this equation were prepared. The plots indicate that the condition of constant error variation along with normal distribution is fairly fulfilled. The value of mean of errors or biasedness for this equation is essentially zero, which is one of the criteria used in selecting our final model. At this stage, it is appropriate to mention that a positive value of biasedness indicates over prediction of the developed model. Similarly, a negative value shows under prediction.

For the verification of the developed model a plot of actual versus predicted values of criterion variables is also drawn. The corresponding plot shows that the equation does not predict well at lower observed values of the criterion variable. For this reason other model structures represented by A-2, A-3, A-4, and A-5 were tried. A-2, is a power model, whereas A-3 was developed to provide more flexibility. Models A-4, and A-5 were tried with some functional forms that account for the value of criterion variable becoming almost constant after a certain age. However, in all of these model structures more or less the same problems of over prediction at lower values of criterion variable was observed.

Finally, a comparison of these different model forms was made. It was found that the prediction of A-5 was better than the other model forms. Therefore, this model form was selected for further analysis. This analysis was based on composite modeling in order to handle the situation of over prediction at lower values of criterion variable.

A turning point was selected at 3.5 million psi. Once this point was selected, the following composite model structure was selected for further analysis:

$$E = ao + a,* [Age/(l - a^2 * Age)] + a^3 * C/F + a * * A/C - a^5 * W/C + a,,* Max + a^7 * Cur$$

If E (calculated from the above Equation) < 3500000 psi, then calculate Ejected by using the following equation.

$$Econrcted = E - a^8[3500000 - E]$$

Statistical Analysis Software (SAS) was used to make an initial estimate of the coefficients of the proposed composite model structures. Later these initial estimates were used by NUMOPT to develop the final Model, given by the following equation.

$$E = 37481300 + 538505000 * [Age/(1 - 17.7942 * Age)] + 14295 * C/F + 281192 * A/C - 61588.5 * W/C + 105851 * Max + 17495.2 * Cur$$

If E (calculated from Equation 4) < 3500000 psi, then calculate ECOIKCKd by using the following equation.

$$Ecorrected = E - 1.0821\,[3500000 - E]$$

In this Model,
E = Dynamic modulus of concrete (psi)
Age = Age of concrete sample (days)
C/F = Coarse aggregate/fine aggregate ratio
A/C = Aggregate (coarse aggregate plus fine aggregate)/cement ratio
W/C = Water/cement ratio
Max = maximum coarse aggregate size (inch)
Cur = Curing time (days)

The value of R2 for the developed Model is 0.808. The values of Se/Sy and relative biasedness for this model are 0.35 and 0.00. The term relative biasedness is defined as the ratio of mean errors to the mean value of criterion variable. The measured values of criterion variable for this model range from 2,768,428.0 to 6,423,247.0 psi.

The plots for errors and their distribution for the developed Model was prepared. These plots indicate that the conditions of constant error variation along with the normal distribution of errors are fairly fulfilled. For further verification, the plot of actual versus predicted values of the criterion variable was also drawn. Which is again helpful for verification of the

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developed Model.

In the developed Model, if we want to see the effect of each predictor variable, it is clear that the value of dynamic modulus increases by increasing Age, C/F, A/C, Max, and Cur. However, W/C has an inverse effect on the 'E' value.

As far as the verification of these results is concerned, Neil (1987) has stated that by increasing C/F, A/C, and Max, the dynamic modulus of concrete increases. The dynamic modulus of concrete also increases with an increase in age and curing time at early stages of hardening due to completion of the hydration process (Samarai, 1983).

As mentioned before, in the developed Model, the dynamic modulus decreases with the increase in W/C ratio due to voids or space that are left after excess water has been removed. The percentage of these voids, which controls the properties of concrete, is directly related to the W/C ratio. Therefore, as the W/C ratio increase, the percentage of voids increases and thus the strength decreases.

At this stage it is important to mention that in the early stage of hardening the temperature also has some effect on the '£' value (Neville, 1987). However, due to availability of only one curing room all the work of this study was performed at room temperature.

5 Sensitivity Analysis Of The Final Model

Sensitivity analysis is a powerful tool to understand the system and the relative importance of the predictor variables. In simple linear or power models, the relative importance is generally assessed by the magnitude of their corresponding R2. However, this criteria does not work for polynomial, exponential, or complex model structures. One of the possible solutions to nonlinear model forms is to generate data that reflects the sensitivity of criterion variable relative to predictor variables. This was applied in the developed Model. In this model the effect of each predictor variable was found by keeping the other variables at some constant value. For simplicity, the mean value of each predictor variable was used for this purpose. These mean values along with all the other selected values of each predictor variable are given in Table 5.

Using the developed model, a data set was generated by changing each predictor variable from minimum to maximum, while keeping the other variables constant. Table 6 presents these results, which indicate that under these conditions, the W/C ratio is the most important variable of the developed Model. A/C is slightly important. Age is slightly important for low values of age. C/F, Max, and cur are unimportant. These results are consistent with R2 values of the linear model presented in Table 4 and are thus helpful for the verification of the final Model.

In order to plot the values of Table 6 the values of predictor variables were normalized by using the minimum value of each predictor variable as the zero point and the maximum value set as 100 percent. Values within the range were proportioned linearly and expressed as a percentage. It is clear that the dynamic modulus increases with age, however, after a certain age it becomes almost constant, which again suggests that the model structure used in the development of the final Model was correct.

6 Conclusions

The developed model of this work is rational, reliable and unbiased. It can be used for the determination of 'E', at any age (up to 56 days) and curing time, with a known mixed design. This model is helpful in reaching a mixed design quickly wit out making and testing many samples ofdifferent mixed proportions, as recommended by ACI Standards 211.1. Since age is one of thepredictor or independent variable in this model, it is helpful in eliminating undue construction delays of newly constructed structural members.

TABLE 5
Mean, Minimum, Maximum and Other Data Points for Predictor Variables of The Developed Model

PREDICTOR VARIABLE	MEAN	MINIMUM	MAXIMUM	OTHER DATA PPOINTS
AGE	24.59	4.00	56.0	7,14,28,37,50
C/F	2.000	1.00	3.00	2
A/C	4.167	3.00	5.50	4
w/c	58.16	48.0	75.0	55,62
Max	0.953	0.50	1.50	0.75, 1
Cur	7.295	2.00	12.0	5

TABLE 6
RESULT OF RELATIVE IMPORTANCE FOR ALL PREDICTORS VARIABLES OF THE DEVELOPED MODEL

PREDICTOR VARIABLE	CALCULATED DYNAMIC MODULUS AT MIN. VALUE OF VARIABLE	CALCULATED DYNAMIC MODULUS AT MAX. VALUE OF VARIABLE
AGE	4634874.0	5035713.0
C/F	4982497.0	5011087.0
A/C	4667797.0	5370777.0
w/c	5622531.0	3959641.0
Max	4948841.0	5054692.0
Cur	4904067.0	5079019.0

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